Reliability Verification, Testing, and Analysis in Engineering Design

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The writing of this book continues a long tradition at Wayne State University (WSU). B. Epstein (1948, 1954, 1958, 1960), a mathematics professor at WSU, pioneered much of the early development of practical life models based on the exponential and extreme value distributions. Later on, Kapur and Lamberson (1977) authored a very popular textbook on engineering reliability that continues to be in wide use today, despite the fact that it emphasizes the use of linear estimation schemes, which are no longer in use. To continue this long tradition, I have written a book that I believe accurately captures the theory and practice behind many of the design verification techniques used in industry.

This book was inspired by my unique opportunity to live and work in metropolitan Detroit, the motor capital of the world. This has afforded me some real-world experiences that I have enjoyed, such as the opportunity to consult with automakers and their tier-1 suppliers. When designing test verification plans, I have seen engineers panic when they realize they do not have the necessary training and background to decide how many items should be placed on test. My motivation for writing this book is based on the need to provide reliability engineers with a reference book that can help them meet these challenges.

This book can be used in the classroom to expose students to the theory and practice of applied life data analysis, or it can be used as a reference book for practicing reliability engineering professionals and their counterparts. Consultants and academicians working in allied areas are also apt to use this book as a
reference. This text contains numerous worked-out examples using either Microsoft® Excel or MINITAB™ statistical computing software, along with a brief list of suggested exercises at the end of each chapter. The former product was chosen for its obvious, ubiquitous nature. The latter was chosen because it has become a very popular choice in the classroom and among Fortune 500 companies who wish to quickly and easily analyze small industrial data sets. The reader is not required to use these products, however, as they are presented only to demonstrate the underlying implementation of efficient computer-based procedures. In keeping with the spirit of this text, the reader might be amazed to find that reference tables for looking up normal probabilities do not appear in this book. The reader is encouraged to use the built-in routines of Excel and similar software to look up these values. Numerous examples appear throughout the book, which should serve to assist the reader in locating these values.

Some of the features and design formulas presented in this text are unique and some are just unusual:

- In the introductory chapter, an overview of modern reliability thinking of the late twentieth and early twenty-first centuries is presented, including emphasis on understanding what is a failure, the importance of understanding customer usage profiles, and the deployment of reliability throughout the product design process from cradle to grave. Although this chapter cannot serve as a comprehensive overview of reliability management techniques, I am certain the reader will find this information useful. Additional topics include the use of Quality Function Deployment for reliability planning, FMEA/FMECA, and the use of DVP&R and its relationship with FMEA.

- The book makes use of Microsoft® Excel spreadsheet and Tool > Solver and Goal Seek nonlinear search procedures wherever possible. Actual spreadsheets are reproduced along with background information on how the procedures are to be run. The book demonstrates how Excel can be used to develop both Fisher matrix and likelihood ratio estimates of reliability metrics. In the latter case, the use of maximum likelihood estimation techniques for developing asymptotic properties is clearly explained.

- MINITAB™ is used to develop Monte Carlo interval estimates of reliability metrics. This is not new, but the reader will find very few textbooks that cover the practical use of Monte Carlo to this extent. The book shows how simple macros can be written in MINITAB™ to run Monte Carlo estimation routines.

- The theory behind the use of rank estimators and the development of small sample binomial confidence limits for success-failure testing is presented to the user in the Appendices to Chapters 2 and 6. The
formulas are shown to be different. Their differences have never before been published in a textbook! The equivalence between Kaplan–Meier, other product-limit estimators including the modified Kaplan–Meier, and Johnson’s formula for estimating the rank of multiply censored data is also demonstrated. This work has also never appeared in a textbook before.

- I closely follow the recommendations by Abernethy (1996), who advises the use of inverse rank regression techniques for estimating the parameters of a distribution (from probability plots). This is not the method usually recommended; however, we now see that the latest release of MINITAB™ (version 13) performs rank regression this way!
- One of the most comprehensive and concise discussions of goodness-of-fit methodology, including the use of ordinary regression $R^2$ statistics to assess fit, is provided. The book also describes how to identify mixtures and competing failure modes from the examination of probability plots, and introduces the use of mixture distributional forms by Tarum (1996) for modeling reliability bathtub phenomena.
- Chapter 6 presents formulas for extended bogey testing, which are usually presented in a more theoretical context. It also discusses the use of tail testing techniques and fully describes the use of a Bayesian adjustment, which permits a sample size reduction of one.
- The book surveys both the use of accelerated life techniques for modeling life versus stress relationships and the use of HALT/HASS (see Chapter 7) for quickly identifying design deficiencies. In the former case, we show how to use MINITAB™ built-in accelerated life-testing routines for modeling the Arrhenius relationship using maximum likelihood techniques for estimating the parameters of the Arrhenius model.
- The book attempts to provide balance by surveying the use of computer-aided engineering techniques for design verification. The uses of finite element models, probabilistic design, etc., are surveyed. The reader is made aware that in the future testing will be used exclusively for confirmation, not for troubleshooting deficient designs!
- Chapter 10 explains the use of simple quantile–quantile (Q–Q) plots for examining differences between life data sets and for estimating acceleration factors.

The coverage of theory is intentionally varied. For example, in Chapter 3, I introduce basic foundations of distributional modeling, including the use of the Z-transform for developing estimates of normal fractiles. This is done to provide some background and reference material to a broad audience of reliability professionals and students. Much of the more advanced material is placed in appendices, and the more advanced material on likelihood estimation is post-
poned until Chapter 9. Everyone should find something to gain from this book, and our less experienced readers will hopefully find expanded uses for this text as they continue to progress.

Finally, I wish to thank the many informal, formal, and anonymous people who have reviewed this text. In particular, I wish to acknowledge the following people: Dave Deepak, Harley-Davidson; Dr. Yavuz Goktas, Baxter Healthcare Corp.; Ron Salzman, Ford Motor Company; James McLinn, Rel-Tech; Leonard Lamberson, Western Michigan University; and the students of Reliability Engineering class, IE 7270, Fall Semester 1999, Wayne State University.

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